Decoherence-free creation of atom-atom entanglement in cavity via fractional adiabatic passage

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We propose a robust and decoherence insensitive scheme to generate controllable entangled states of two three-level atoms interacting with an optical cavity and a laser beam. Losses due to atomic spontaneous transitions and to cavity decay are efficiently suppressed by employing fractional adiabatic passage and appropriately designed atom-field couplings. In this scheme the two atoms traverse the cavity-mode and the laser beam in opposite directions as opposed to other entanglement schemes in which the atoms are required to have fixed locations inside a cavity. We also show that the coherence of a traveling atom can be transferred to the other one without populating the cavity-mode.

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The physics of entanglement provides the basis of applications such as quantum information processing and quantum communications. Particles can then be viewed as carriers of quantum bits of information and the realization of engineered entanglement is an essential ingredient of the implementation of quantum gates [1], cryptography [2] and teleportation [3]. The creation of long-lived entangled pairs of atoms may provide reliable quantum information storage. The idea is to apply a set of controlled coherent interactions to the atoms of the system in order to bring them into a tailored entangled state. The problem of controlling entanglement is thus directly connected to the problem of coherent control of population transfer in multilevel systems.

In the context of cavity QED, one of the main obstacles to realize atom-atom entanglement is the decoherence resulting from the cavity decay. Additionally, the cavity couples to an excited state of the atom that undergoes spontaneous emission. Regarding these considerations, in recent years several schemes to entangle atoms [4, 5] and to implement quantum gates [6, 7, 8, 9, 10] using optical cavities have been proposed. To avoid decoherence effects, it is most convenient to design transfer strategies that do not populate the decoherence channels during the time-evolution of the system.

In this paper we propose an alternative way to entangle two traveling atoms interacting with an optical cavity and a laser beam, based on 3-level interactions in a Λ-configuration. This method is based on the coherent creation of superposition of atom-atom-cavity states via fractional stimulated Raman adiabatic passage (f-STIRAP) [11], that keeps the cavity-mode and the excited atomic states unpopulated during the whole interaction. In Ref. [12], using f-STIRAP and a one-atom dark state, a robust scheme to generate atom-atom entanglement was proposed, where the two traveling atoms encounter the cavity-mode one by one. Here, the two atoms enter simultaneously into the cavity in such a way that the system follows a two-atom dark state that allows us to keep additionally the cavity-mode empty.

We consider the situation described in Fig. 1, where the two atoms move in planes orthogonal to the z axis as follows:

$$z_1 = z_0, \ \ x_1 = -x_0 + v_1 t \cos\theta_1, \ \ y_1 = -y_0 + v_1 t \sin\theta_1,$$
$$z_2 = 0, \ \ x_2 = x_0 + v_2(t - \tau)\cos\theta_2,$$
$$y_2 = -y_0 + v_2(t-\tau)\sin\theta_2, \tag{1}$$

where $(x_i, y_i, z_i), i = 1, 2$ are the coordinates of the i-th atom $(x_0, y_0 > 0)$, τ is the time-delay of the second atom with respect to the first one, θ_i is the angle that the i-th atom constructs with the positive direction of the x axis ($\theta_1 \in [0, \pi/2[$, $\theta_2 \in]\pi/2, \pi]$), and v_i is the velocity of the i-th atom. Since the laser field propagates in direction of the y axis, the time-dependent optical phase of the laser field seen by each atom is

$$\varphi_1(t) = \omega_L t - k v_1 t \sin\theta_1, \quad \varphi_2(t-\tau) = \omega_L t - k v_2 (t-\tau)\sin\theta_2, \tag{2}$$

where k is the wavevector magnitude of the laser field. Figure 2 represents the linkage pattern of the atom-cavity-laser system. The laser pulse associated to the Rabi frequency $\Omega(t)$ couples the states $|g_1\rangle$ and $|e\rangle$, and the cavity-mode with Rabi frequency $G(t)$ couples the states $|e\rangle$ and $|g_2\rangle$. The Rabi frequencies $\Omega(t)$ and $G(t)$ are chosen real without loss of generality. These two fields interact with the atom with a time
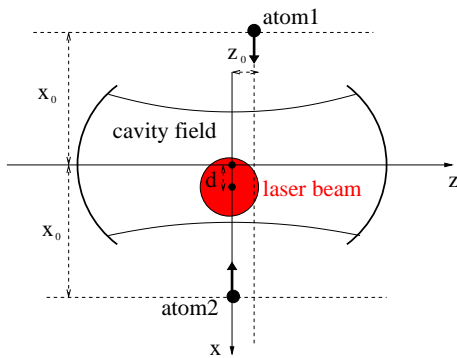

FIG. 1: Geometrical configuration of the atoms-cavity-laser system in the proposed scheme. The propagation direction of the laser beam is parallel to y axis and perpendicular to the page.

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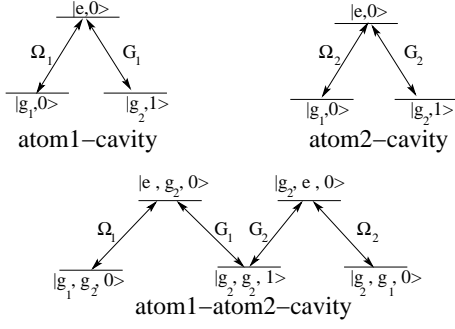‡Electronic address: jauslin@u-bourgogne.fr

FIG. 2: Linkage pattern of the system corresponding to the effective Hamiltonian.

delay, each of the fields is in one-photon resonance with the respective transition. The semiclassical Hamiltonian of this system in the resonant approximation where

$$|\Omega_0|, |G_0| \ll \omega_e, \omega_C, |\partial\varphi_1/\partial t|, |\partial\varphi_2/\partial t|, \quad (3)$$

with Ω_0, G_0 the peak values of the Rabi frequencies, can then

be written as ($\hbar = 1$)

$$
\begin{aligned}
H(t) &= \sum_{i=1,2} \big[\omega_e|e\rangle_{ii}\langle e| + \big(G_i(t)\, a|e\rangle_{ii}\langle g_2| + \text{H.c.}\big) \\
&\quad + \big(\Omega_i(t)e^{i\varphi_i(t)}|g_1\rangle_{ii}\langle e| + \text{H.c.}\big)\big] + \omega_C a^\dagger a, \quad (4)
\end{aligned}
$$

where the subscript i on the states denotes the two atoms, a is the annihilation operator of the cavity mode, ω_e is the energy of the atomic excited state ($\omega_{g_1} = \omega_{g_2} = 0$), and ω_C is the frequency of the cavity mode taking resonant $\omega_C = \omega_L = \omega_e$ which implies $kv_i \sin\theta_i \ll \omega_L$. In the following we consider the state of the atom1-atom2-cavity system as $|A1, A2, n\rangle$ where $\{A1, A2 = g_1, e, g_2\}$, and $\{n = 0, 1\}$ is the number of photons in the cavity-mode.

Regarding Figure 2, the subspace $\mathcal{S}$ generated by the states $\{|g_1, g_2, 0\rangle, |e, g_2, 0\rangle, |g_2, g_2, 1\rangle, |g_2, e, 0\rangle, |g_2, g_1, 0\rangle\}$ is decoupled under H from the rest of the Hilbert space of the system. If we consider the initial state of the system as $|g_1, g_2, 0\rangle$, the Hamiltonian of the system in the subspace $\mathcal{S}$ will be

$$
H_P(t) := PHP = \begin{pmatrix}
0 & \Omega_1(t)e^{+i\varphi_1(t)} & 0 & 0 & 0 \\
\Omega_1(t)e^{-i\varphi_1(t)} & \omega_e & G_1(t) & 0 & 0 \\
0 & G_1(t) & \omega_C & G_2(t-\tau) & 0 \\
0 & 0 & G_2(t-\tau) & \omega_e & \Omega_2(t-\tau)e^{-i\varphi_2(t-\tau)} \\
0 & 0 & 0 & \Omega_2(t-\tau)e^{+i\varphi_2(t-\tau)} & 0
\end{pmatrix}, \quad (5)
$$

where P is the projector on the subspace $\mathcal{S}$. The effective Hamiltonian is thus given by

$$
H^{\text{eff}} := R^\dagger H_P R - iR^\dagger \frac{\partial R}{\partial t} = \begin{bmatrix}
0 & \Omega_1(t) & 0 & 0 & 0 \\
\Omega_1(t) & kv_1\sin\theta_1 & G_1(t) & 0 & 0 \\
0 & G_1(t) & kv_1\sin\theta_1 & G_2(t-\tau) & 0 \\
0 & 0 & G_2(t-\tau) & kv_1\sin\theta_1 & \Omega_2(t-\tau) \\
0 & 0 & 0 & \Omega_2(t-\tau) & k(v_1\sin\theta_1 - v_2\sin\theta_2)
\end{bmatrix}, \quad (6)
$$

where the unitary transformation R is

$$
R = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & e^{-i\varphi_1(t)} & 0 & 0 & 0 \\
0 & 0 & e^{-i\varphi_1(t)} & 0 & 0 \\
0 & 0 & 0 & e^{-i\varphi_1(t)} & 0 \\
0 & 0 & 0 & 0 & e^{-i[\varphi_1(t)-\varphi_2(t-\tau)]}
\end{pmatrix}. \quad (7)
$$

The dynamics of the system is governed by the Schrödinger equation $i\frac{\partial}{\partial t}|\Phi(t)\rangle = H^{\text{eff}}(t)|\Phi(t)\rangle$.

An essential condition for the STIRAP and f-STIRAP processes is the four-photon resonance between the states $|g_1, g_2, 0\rangle$ and $|g_2, g_1, 0\rangle$ which means:

$$|\Delta := k(v_1\sin\theta_1 - v_2\sin\theta_2)| \ll |\Omega_0|, |G_0|. \quad (8)$$

This condition can be achieved by control of the velocity of atoms, and of the deflection angles of the atoms $\theta_{i=1,2}$. Nu-

merics shows that, in practice, the condition (8) is satisfied for $\Delta \lesssim \{\Omega_0, G_0\}/100$. Assuming this condition allows one to consider $\Delta \sim kv_1\sin\theta_1 \sim 0$ in Eq. (8), if we additionally assume $v_1 \sim v_2 \equiv v$. In this case τ is the delay of arrival at the center of the cavity ($x = y = 0$) of the second atom with respect to the first one.

The system is taken to be initially in the state $|g_1, g_2, 0\rangle$,

$$|\Phi(t_i)\rangle = |g_1, g_2, 0\rangle. \quad (9)$$

The goal is to transform it at the end of interaction into an atom-atom entangled state

$$
\begin{aligned}
|\Phi(t_f)\rangle &= \cos\vartheta|g_1, g_2, 0\rangle + \sin\vartheta|g_2, g_1, 0\rangle \\
&= (\cos\vartheta|g_1, g_2\rangle + \sin\vartheta|g_2, g_1\rangle)|0\rangle, \quad (10)
\end{aligned}
$$

where ϑ is a constant mixing angle ($0 \leq \vartheta \leq \pi/2$), and the cavity-mode state factorizes and is left in the vacuum

state. The qubits are stored in the two degenerate ground states of the atoms. The decoherence due to atomic spontaneous emission is produced if the states $\{|e, g_2, 0\rangle, |g_2, e, 0\rangle\}$ are populated, and the cavity decay occurs if the state $|g_2, g_2, 1\rangle$ is populated during the adiabatic evolution of the system. Therefore we will design the Rabi frequencies $\{\Omega_1(t), G_1(t), \Omega_1(t), G_2(t)\}$ in our scheme such that these states are not populated during the dynamics.

One of the instantaneous eigenstates (the two-atom dark state) of $H^{\mathrm{eff}}(t)$ which corresponds to a zero eigenvalue is [10]

$$\begin{aligned} |D(t)\rangle \;=\; C\Big(& G_1\Omega_2|g_1, g_2, 0\rangle - \Omega_1\Omega_2|g_2, g_2, 1\rangle \\ & + G_2\Omega_1|g_2, g_1, 0\rangle \Big), \end{aligned} \tag{11}$$

where C is a normalization factor. The possibility of decoherence-free generation of atom-atom entanglement arises from the following behavior of the dark state:

$$\lim_{t \to t_i} \frac{\Omega_1(t)}{\Omega_2(t)} = 0, \qquad |D(t_i)\rangle \sim |g_1, g_2, 0\rangle \tag{12a}$$

$$\lim_{t \to t_f} \frac{\Omega_1(t)}{\Omega_2(t)} = \tan\vartheta,$$
$$|D(t_f)\rangle \sim \cos\vartheta|g_1, g_2, 0\rangle + \sin\vartheta|g_2, g_1, 0\rangle \tag{12b}$$

$$t_i < t < t_f, \; G_1(t) \sim G_2(t) \gg \Omega_1(t), \Omega_2(t),$$
$$|D(t)\rangle \sim \Omega_2(t)|g_1, g_2, 0\rangle + \Omega_1(t)|g_2, g_1, 0\rangle. \tag{12c}$$

Equations (12a) and (12b) are known as f-STIRAP conditions [11, 12], and the condition (12c) guarantees the absence of population in the state $|g_2, g_2, 1\rangle$ during the time-evolution of the system [13, 14]. Equation (12b) means that the Rabi frequencies fall off in a constant ratio, during the time interval where they are non-negligible. We remark that this formulation opens up the possibility to implement f-STIRAP with Gaussian pulses. The goal in the following is to show that such a pulse sequence can be designed in a cavity by an appropriate choice of the parameters.

In an optical cavity, the spatial variation of the atom-field coupling for the maximum coupling TEM$_{00}$ mode, resonant with the $|e\rangle \leftrightarrow |g_2\rangle$ atomic transition, is given by

$$G(x, y, z) = G_0 \, e^{-(x^2 + y^2)/W_C^2} \cos\left(\frac{2\pi z}{\lambda}\right), \tag{13}$$

where W_L is the waist of the cavity mode, and $G_0 = -\mu\sqrt{\omega_C/(2\epsilon_0 V_{\mathrm{mode}})}$ with μ and V_{mode} respectively the dipole moment of the atomic transition and the effective volume of the cavity mode. The spatial variation of the atom-laser coupling for the laser beam of Fig. 1 is

$$\Omega(x, z) = \Omega_0 \, e^{-(x^2 + z^2)/W_L^2}, \tag{14}$$

where W_L is the waist of the laser beam, and $\Omega_0 = -\mu\mathcal{E}/2$ with $\mathcal{E}$ the amplitude of the laser field. Figure 1 shows a situation where the first atom, initially in the state $|g_1\rangle$, goes with
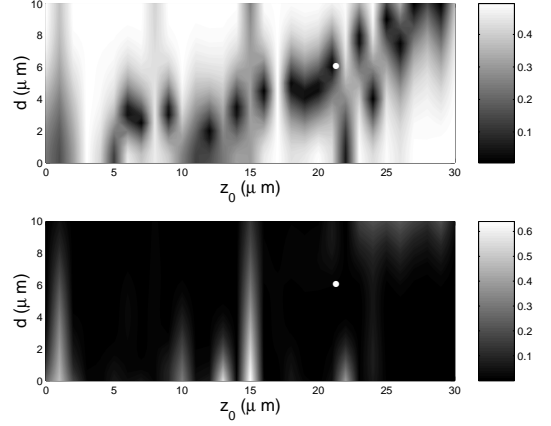

FIG. 3: Top panel: contour plot at the final time t_f of $|\frac{1}{2} - P_{|g_1, g_2, 0\rangle}(t_f)|$ as a function of z_0 and d (black areas correspond to approximately half population transfer) with the pulse parameters as $W_L = 20\mu$m, $W_C = 40\mu$m, $v = 2$ m/s, $\lambda = 780$nm, $\Omega_0 = 20\frac{v}{W_L}$, $G_0 = 100\frac{v}{W_C}$. Bottom panel: The same plot for the sum of the final populations in intermediate states $P_{|e, g_2, 0\rangle}(t_f) + P_{|g_2, g_2, 1\rangle}(t_f) + P_{|g_2, e, 0\rangle}(t_f)$ where black areas correspond to approximately zero population. The white dot shows specific values of z_0 and d used in Fig. 4 to obtain a half-STIRAP process.

velocity v (on the $y = 0$ plane at $z = z_0$ line) through an optical cavity initially in the vacuum state $|0\rangle$ and then encounters the laser beam, which is parallel to the y axis (orthogonal to the cavity axis and the trajectory of the atom). The laser beam is resonant with the $|e\rangle \leftrightarrow |g_1\rangle$ transition. The distance between the center of the cavity and the laser axis is d. The second atom, synchronized with the first one $\tau = 0$, moves with the same velocity v on the $y = 0$ plane at $z = 0$ in the opposite direction with respect to the first atom. The traveling atoms encounter the time-dependent and delayed Rabi frequencies of the cavity-mode and the laser fields as follows

$$G_1(t) = G_0 \, e^{-(vt)^2/W_C^2} \cos\left(\frac{2\pi z_0}{\lambda}\right), \tag{15a}$$

$$\Omega_1(t) = \Omega_0 \, e^{-z_0^2/W_L^2} \, e^{-(vt-d)^2/W_L^2}, \tag{15b}$$

$$G_2(t) = G_0 \, e^{-(vt)^2/W_C^2}, \tag{15c}$$

$$\Omega_2(t) = \Omega_0 \, e^{-(vt+d)^2/W_L^2} \tag{15d}$$

where the time origin is defined when the atoms meet the center of the cavity at $x = y = 0$. The appropriate values of z_0 and d that lead to the f-STIRAP process can be extracted from a contour plot of the final population $P_{|g_1, g_2, 0\rangle}(t_f) := |\langle g_1, g_2, 0|\Phi(t_f)\rangle|^2$ as a function of z_0 and d that we calculated numerically (see Fig. 3). The white dot in Fig. 3 shows values of z_0 and d to obtain an f-STIRAP process with $\vartheta \simeq \pi/4$ (called half-STIRAP). It has been chosen such that at the end of interaction $P_{|g_1, g_2, 0\rangle}(t_f) \simeq P_{|g_2, g_1, 0\rangle}(t_f) \simeq 0.5$, and the populations of the other states of the subspace $\mathcal{S}$ are zero.

Figure 4 shows for parameter values associated to the white dot in Fig. 3, (a) the time dependence of the Rabi frequen-
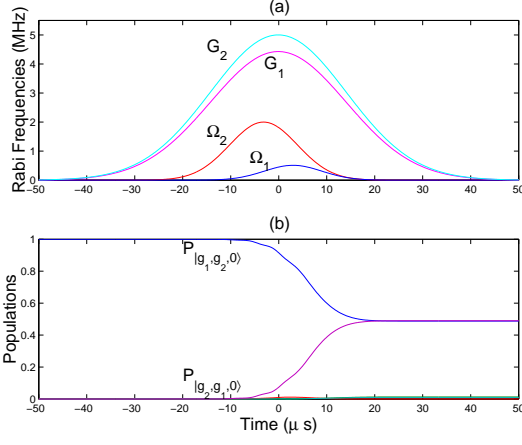
FIG. 4: (a) Rabi frequencies of the cavity-mode and the laser field for two atoms. (b) Time evolution of the populations which represents a two-atom half-STIRAP.
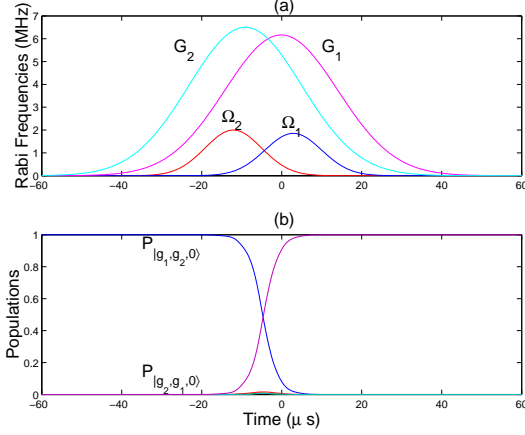

FIG. 5: (a) Rabi frequencies of the cavity-mode and the laser field for two atoms with the pulse parameters as $v = 2$ m/s, $W_L = 20\mu$m, $W_C = 40\mu$m, $\lambda = 780$ nm, $\Omega_0 = 2$ MHz, $G_0 = 6.5$ MHz, $z_0 = 5.5\mu$m, $d = 6\mu$m and $\tau = -9\mu$s. (b) Time evolution of the populations which represents a two-atom STIRAP.

cies of half-STIRAP for two atoms, and (b) the time evolution of the populations which shows that the population

is split at 50% among the states $|g_1, g_2, 0\rangle$, $|g_2, g_1, 0\rangle$ and is almost zero for the other states of $\mathcal{S}$ during the interaction for $G_0 \sim 3\Omega_0$. This case corresponds to the generation of the maximally atom-atom entangled state $1/\sqrt{2}(|g_1, g_2, 0\rangle + |g_2, g_1, 0\rangle)$ by adiabatic passage. Assuming Gaussian pulse profiles for $\Omega(t)$ and $G(t)$ of widths $T_L = W_L/v$ and $T_C = W_C/v$ respectively, the sufficient condition of adiabaticity is $\Omega_0 T_L, G_0 T_C \gg 1$.

The state $|g_2, g_2, 0\rangle$ is stationary state of the system. If the second atom enters inside the cavity before the first one ($\tau < 0$), we can transfer completely the population from the state $|g_1, g_2, 0\rangle$ to $|g_2, g_1, 0\rangle$ *without populating the other states of $\mathcal{S}$ during the dynamics* (see Fig. 5). In particular, given that the first atom initially is prepared in a coherent superposition of the ground states $\alpha|g_1\rangle + \beta|g_2\rangle$, that the second atom is initially in the state $|g_2\rangle$, and that the cavity-mode is initially in the vacuum state, a two-atom STIRAP will coherently map this superposition onto the second atom:

$$\alpha|g_1, g_2, 0\rangle + \beta|g_2, g_2, 0\rangle \rightarrow \alpha|g_2, g_1, 0\rangle + \beta|g_2, g_2, 0\rangle. \quad (16)$$

In summary, we have proposed a robust and decoherence-free scheme to generate atom-atom entanglement, using the f-STIRAP technique in Λ-systems. This scheme is robust with respect to variations of the velocity of the atoms v, of the peak Rabi frequencies G_0, Ω_0 and of the field detunings, but not with respect to the parameters d, z_0, describing the relative positions of the laser beam and the cavity, shown in Fig. 1. Our scheme can be implemented in an optical cavity with $G_0 \sim \kappa \sim \Gamma$. The necessary condition to suppress the cavity decoherence is $G_0 \gg \Omega_0$ which is satisfied in practice for $G_0 \sim 3\Omega_0$ (see Figs. 4,5). For given values of W_C, W_L, the adapted values of d and z_0 in the f-STIRAP process can be determined from a contour plot of the final populations. Decoherence channels are suppressed during the whole evolution of the system. In this scheme, as opposed to the scheme of Ref. [10], we do not need to fix the atoms inside the optical cavity nor to apply two laser beams for each of the individual atoms.

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